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Mixed convection flow along a vertical plate subjected to time-periodic surface temperature oscillations

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Abstract

The effect of the periodic oscillations of the surface temperature with time on the mixed convection from a vertical plate is investigated. The problem has been simplified by employing the laminar boundary layer and Boussinesq approximations. The fully implicit finite-difference scheme is used to solve the dimensionless governing equations. The results for the laminar flow of air having $Pr = 0.72$ and water having $Pr = 7.00$ are presented for an isothermal flat plate and for the periodic oscillation of the temperature on the plate. The results show the steady periodic variation of the Nusselt number and friction coefficient for both aiding and opposing flows with different amplitudes and frequencies of the oscillating surface temperature. For both aiding and opposing flow, both the Nusselt number and the friction coefficient oscillate with the oscillation of the plate temperature and for some cases (at high amplitudes and frequencies) the Nusselt number becomes negative. For constant Prandtl number and Richardson number, the cyclic average values of the Nusselt number are decreasing with increasing either the amplitude or the frequency of the surface temperature oscillations. The cyclic average values of the friction coefficient, for all the cases considered, are found to be constant and approximately equal to the values for non-oscillating surface temperature.

Keywords: Mixed convection; Periodic convection; Boundary layer; Oscillating temperature; Numerical study

1. Introduction

Mixed (combined free and forced) convection from a vertical flat plate is important in nature and in many practical transport process devices, such as furnaces, electronic devices cooled by external forced circulation, solar collectors, chemical processing equipments and others. In these applications and others, the change in the plate temperature causes free and/or forced convection flow could be a sudden change or a periodic one, leading to a variation in the flow. The literature shows that the steady and transient mixed convection theory is well established and has been investigated by various researchers. Representative studies in this area may be found in the books by Gebhart et al. [1], Bejan [2], Pop and Ingham [3] and Cebeci [4].

The theory of mixed convection shows that the ratio of the Grashof number to the square of Reynolds number (Gr/Re^2) which is known as the Richardson number, has a great influence on the flow for a constant plate temperature. The forced convection is dominating for small values of the Richardson number while free convection takes over for large values of Richardson number. Steady mixed convection from a vertical plate has been studied by various authors, including Merkin [5], Wilks [6], Lloyd and Sparrow [7], Gryzagoridis [8] and Jaluria [9]. A combination of series expansion and numerical integration was used by Merkin [5] to solve the mixed convection problem along an isothermal wall for both aiding (the external flow is in the same direction as the buoyancy force) and opposing (the external flow is in the opposite direction to the buoyancy force) flows for a Prandtl number of unity. Merkin [5] has shown that in the case of opposing flow, as buoyancy effects increase,

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Nomenclature

$A(\Re)$	amplitude of $\Re$, Eq. (16)
C_f	friction coefficient, Eq. (13)
g	magnitude of the gravitational acceleration . . . . $m \cdot s^{-2}$
Gr	Grashof number based on L
L	plate height . . . . m
Nu_x	local Nusselt number, Eq. (14)
Pr	Prandtl number
Re	Reynolds number based on L
Ri	Richardson number, $= Gr/Re^2$
t	time . . . . s
T	temperature . . . . K
u, v	velocity components . . . . $m \cdot s^{-1}$
U, V	non-dimensional velocity components
x, y	Cartesian coordinates . . . . m
X, Y	non-dimensional Cartesian coordinates

Greek symbols

α	thermal diffusivity . . . . $m^2 \cdot s^{-1}$
β	coefficient of volume expansion . . . . K^{-1}
ε	non-dimensional amplitude
ν	kinematic viscosity . . . . $m^2 \cdot s^{-1}$
θ	non-dimensional temperature
τ	non-dimensional time
ω	frequency . . . . s^{-1}
Ω	non-dimensional frequency
$\Re$	either $Nu_x/\sqrt{Re}$ or $C_{fx}\sqrt{Re}$

Subscripts

w	wall
∞	free stream

the boundary layer separates from the plate at a Richardson number $= 0.192357$. A similar study has been carried out by Wilks [6] and the results were obtained again for $Pr = 1$ but for a constant heat flux boundary condition. Lloyd and Sparrow [7] have obtained a complete solution to the aiding mixed convection from a vertical isothermal flat plate by using the local similarity method for different values of the Prandtl number $Pr = 0.003$, 0.01, 0.03, 0.72, 10 and 100 and for Richardson number in the range 0 to 4. The aiding mixed convection from a vertical plate has been investigated experimentally by Gryzagoridis [8]. Experimental measurements of the Nusselt number of air ($Pr = 0.72$) are presented by Gryzagoridis [8] for a wide range of Richardson number (0 to 500). A numerical study of aiding mixed convection flow over localized, finite-sized isoflux sources located on a vertical adiabatic vertical surface has been carried out by Jaluria [9]. In this study, the heat transfer coefficient for the upper source is found to depend very strongly on the separation distance between the sources, and increasing this distance leads to an increase in the heat transfer coefficient.

The development in the theory and applications of the mixed convection from the vertical plate has led to an increased interest in the transient and unsteady mixed convection flows. Sammakia et al. [10] studied the transient response of the aiding mixed convection from a vertical flat surface with both a uniform surface temperature and uniform heat flux from the surface for $Pr = 0.72$ and $Pr = 7.6$. They used the explicit finite difference method to solve the dimensionless governing equations and presented the results for the velocity and temperature profiles at different time steps until the steady state is reached. In another study by the same group, Sammakia et al. [11] measured and calculated transient mixed convection from a vertical flat surface with uniform heat flux from the surface for $Pr = 0.72$. A similar transient mixed convection study with different mathematical formulation, using the model given by Yan [12], was carried out by Mai et al. [13] for uniform heat flux boundary condition. Mai et al. [13] have employed the implicit finite-difference scheme to solve the governing equations for both transient aiding and opposing flows and present the velocity and temperature profiles for different time steps until the steady state is reached. Zubair and Kadaba [14] have constructed similarity variables of the transient mixed convection problem and obtained new solutions. Merkin and Pop [15] and Steinruck [16] have shown that the similarity solutions of the boundary-layer flow equations describing mixed convection flow along a vertical plate exists if the difference between the temperature of the plate and the ambient temperature is inversely proportional to the distance from the leading edge of the plate. The effect of wall conduction on the characteristics of unsteady mixed convection is important in the engineering applications. These types of unsteady conjugated mixed convection problems were studied numerically by Yan and Lee [17] and Lee and Yan [18] for vertical channel flows.

It is noted that for steady state and transient mixed convection problems, the constant, streamwise surface temperature variation or constant heat flux are usually assumed in the above mentioned studies. However, in industrial applications, quite often the convection heat transfer is a periodic process. Saeid [19] has studied the periodic free convection from a vertical plate. The laminar boundary layer theory is used to study the effect of periodic plate temperature oscillations with different amplitudes and frequencies on the free convection flow with different Prandtl numbers. Recently Saeid and Mohamad [20] have considered the surface temperature oscillation in free convection from a vertical plate immersed in porous media. In the above two studies, [19] and [20], it is found that the free convection heat transfer from the vertical plate decreases with an increase in either the amplitude or frequency of the surface temperature oscillation. In this paper the problem studied by Saeid

[19] for free convection is extended to mixed convection for both aiding and opposing flows of air ($Pr = 0.72$) and water ($Pr = 7.00$).

2. Mathematical model

A schematic diagram of the physical model and coordinate system under study in the present paper is shown in Fig. 1 for aiding flow. It may be noted that in the case of opposing flow the direction of the gravitational force is opposite and it is negligible for the forced convection flow.

The governing equations of the present problem are the continuity, momentum and energy. They are coupled elliptic equations, and therefore of considerable complexity. To overcome such difficulty, the boundary layer and Boussinesq approximations are used with the absence of heat generation and viscous dissipation. The resulting two-dimensional equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where the vertical flat plate is considered to be along x-axis and the y-axis is normal to the plate. In accordance with the present problem, the boundary conditions are as follows:

$$u(0, y, t) = U_\infty, \qquad v(0, y, t) = 0$$
$$T(0, y, t) = T_\infty \tag{4a}$$

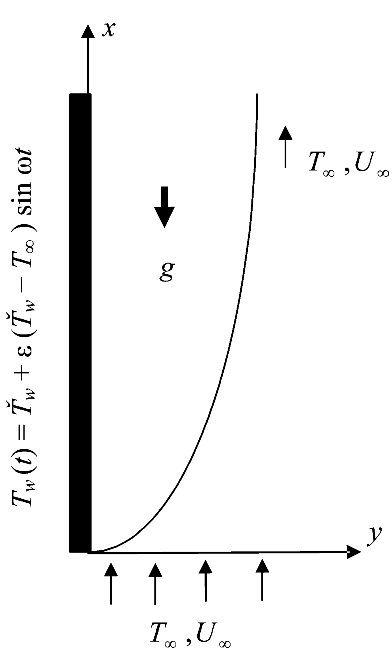


Fig. 1. Schematic diagram of the physical model and coordinate system.

$$u(x, 0, t) = 0, \qquad v(x, 0, t) = 0$$
$$T(x, 0, t) = T_w(t) \tag{4b}$$
$$u(x, \infty, t) = U_\infty, \qquad \partial v(x, \infty, t)/\partial y = 0$$
$$T(x, \infty, t) = T_\infty \tag{4c}$$

It is assumed that the laminar boundary layer is developed without heat transfer $\{T_w(t) = T_\infty\}$ before heating the plate and mixed convection process taking place. Therefore the initial condition is calculated from solving Eqs. (1) and (2) for steady state and zero buoyant term in Eq. (2) with the boundary conditions (4).

The plate temperature condition is assumed to oscillate periodically over an average value $\check{T}_w$ with amplitude ε and frequency ω. Therefore the following plate temperature condition is used:

$$T_w(t) = \check{T}_w + \varepsilon(\check{T}_w - T_\infty)\sin\omega t \tag{5}$$

In order to simplify the problem and to generalize the results, the above equations are written in a non-dimensional form by employing the following boundary layer dimensionless variables:

$$X = x/L, \qquad Y = \frac{y}{L}Re^{1/2}$$
$$\tau = tU_\infty/L, \qquad \Omega = \omega L/U_\infty$$
$$U = u/U_\infty, \qquad V = \frac{v}{U_\infty}Re^{1/2}$$
$$\theta = \frac{T - T_\infty}{\check{T}_w - T_\infty} \tag{6}$$

where L is the plate height and $Re = U_\infty L/\nu$ is the Reynolds number based on the characteristic length L. Using (6), the governing equations reduce to the following non-dimensional boundary layer equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \frac{Gr}{Re^2}\theta \tag{8}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{9}$$

where $Gr = g\beta L^3\Delta T/\nu^2$ is the Grashof number based on the characteristic length L and average temperature difference $\Delta T = \check{T}_w - T_\infty$ and $Pr = \nu/\alpha$ is the Prandtl number. Using the dimensionless variables (6), the dimensionless boundary conditions (4) become:

$$U(0, Y, \tau) = 1, \qquad V(0, Y, \tau) = 0$$
$$\theta(0, Y, \tau) = 0 \tag{10a}$$
$$U(X, 0, \tau) = 0, \qquad V(X, 0, \tau) = 0$$
$$\theta(X, 0, \tau) = \theta_w(\tau) = 1 + \varepsilon\sin(\Omega\tau) \tag{10b}$$
$$U(X, \infty, \tau) = 1, \qquad \partial V(X, \infty, \tau)/\partial Y = 0$$
$$\theta(X, \infty, \tau) = 0 \tag{10c}$$

The initial condition is calculated from solving Eqs. (7) and (8) for steady state and zero buoyant term in Eq. (8) with the boundary conditions (10).

3. Numerical scheme

The momentum equation (8) and energy equation (9) are solved using the integration over a control volume based on the fully implicit scheme which is unconditionally stable. The power-law scheme is used for the convection–diffusion formulation [21]. Finally, the finite-difference equation corresponding to the continuity equation (11) is developed using the expansion point $(i+1, j-\frac{1}{2})$, where i and j are the indices along X and Y respectively [22]. The resulting finite-difference equation is given by:

$$V_{i+1,j} = V_{i+1,j-1} - \frac{\Delta Y_w}{2(\Delta X_n)}(U_{i+1,j} + U_{i+1,j-1} - U_{i,j} - U_{i,j-1}) \tag{11}$$

where ΔY_w and ΔX_n are the grid spaces between the nearest west and north of the (i, j) points and the point (i, j) itself respectively. The solution domain, therefore, consists of grid points at which the discretization equations are applied. In this domain X by definition varies from 0 to 1. However, the choice of the value of Y, corresponding to $Y = \infty$, has an important influence on the solution. The effect of different values to represent $Y = \infty$ on the numerical scheme has been investigated and it is concluded that the value of $Y = 7$ is sufficiently large. Further larger values of Y produced the results with indistinguishable difference. The stretched grid has been selected in both the X and Y directions such that the grid points clustered near the plate and near the leading edge of the flat plate as there are steep variations of the velocity and temperature in these regions. The algorithm needs iteration for the coupled equations (7)–(9). The iteration continues to solve for the new dependent variables values using the previous iteration values until the iterative converged solution is obtained. The convergence condition used for the three dependent variables θ, U, and V is

$$\mathrm{Max}\left|\frac{\varphi^n - \varphi^{n-1}}{\varphi^n}\right| < 10^{-3} \tag{12}$$

where φ is the general dependent variable and the superscript n represents the iteration step number. The time increment is $\Delta\tau = 0.01$ for the isothermal plate case, and $\Delta\tau = 2\pi/(1000\Omega)$ for the oscillation surface temperature case. It is found that smaller than these values have no significant difference in the results. It is found that the mesh size of (100×30) gives acceptable accurate results, which is used in the present study. Moreover, the grid independence test is performed for the mesh size of (200×60) and the results are almost similar. The differences in $C_{fx}\sqrt{Re}$ and $Nu_x/\sqrt{Re}$ is less than 0.2%.

The initial condition is generated as mentioned earlier from solving Eqs. (7) and (8) for steady state and zero buoyant term in Eq. (8) with the boundary conditions (10). The results are compared with the Blasius similarity solution of the laminar boundary layer flow at the upper end of the plate $X = 1$ for both velocity components U and V together with the friction coefficient. The friction coefficient is defined as follows:

$$C_{fx} = \frac{\tau_w}{\rho U_\infty^2} = \frac{\rho\nu(\partial u/\partial y)_{y=0}}{\rho U_\infty^2} \tag{13a}$$

i.e.

$$C_{fx}\sqrt{Re} = \left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{13b}$$

The solution is marched in time until the steady state is obtained for Eqs. (7) and (8) and it is found that the maximum discrepancy in the velocity components and $C_{fx}\sqrt{Re}$ is less than 0.3% from the Blasius similarity solution. The value of $C_{fx}\sqrt{Re} = 0.3311$ is found and the Blasius solution is $C_{fx}\sqrt{Re} = 0.332$.

4. Results and discussion

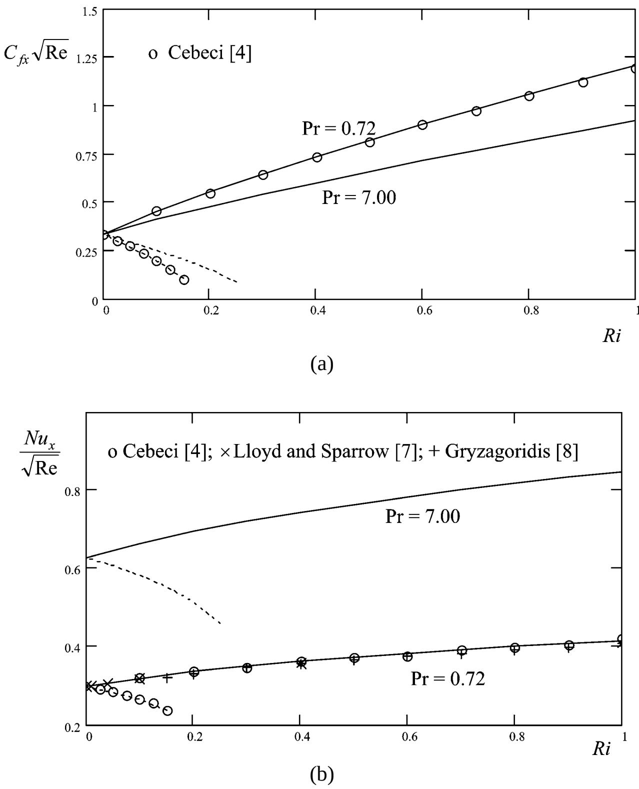
The algorithm explained in the previous section is first tested before studying the effect of surface temperature oscillation on the mixed convection from vertical plate. A step change in the surface temperature is considered and the surface temperature increases suddenly from ambient to the average surface temperature $\overline{T}_w$ (i.e., $\varepsilon = 0$). The local Nusselt number is calculated for each time step at the upper end of the plate $X = 1$, where the local Nusselt number is defined in the present study as follows:

$$Nu_x = \frac{-k(\partial T/\partial y)_{y=0}}{T_w(t) - T_\infty}\frac{x}{k} \tag{14a}$$

or

$$\frac{Nu_x}{\sqrt{Re}} = \frac{-X}{\theta_w(\tau)}\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \tag{14b}$$

where k is the thermal conductivity of the fluid and $\theta_w(\tau) \neq 0$ where the present problem arises. The iteration for solving Eqs. (7)–(9) will continue until the steady state solution is obtained, where the values of $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ become constant. The steady state results of $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ are compared with the numerical results of Lloyd and Sparrow [7] and Cebeci [4] and with the experimental results of Gryzagoridis [8] for $Pr = 0.72$. Fig. 2 shows the variation of $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ with Richardson number for both aiding (heated surface) and opposing (cooled surface) flows for air ($Pr = 0.72$) and water ($Pr = 7.00$). For water ($Pr = 7.00$), the steady state value of Nusselt number in the forced convection mode ($Ri = 0$) is found $Nu_x/\sqrt{Re} = 0.6267$ comparing with the similarity solution ($Nu_x = 0.332 Pr^{1/3} Re_x^{1/2}$) or $Nu_x/\sqrt{Re} = 0.6351$. The difference is around 1.3%.



(b)

Fig. 2. Steady state variation of (a) $C_{fx}\sqrt{Re}$, and (b) $Nu_x/\sqrt{Re}$ with the Richardson number (— aiding flow, ····· opposing flow) for $\varepsilon = 0$.

It is found that there is no converged solution for the opposing flow with $Ri \geqslant 0.2$ for air with $Pr = 0.72$ and the flow in such cases are complex where separation takes place ($C_{fx}\sqrt{Re} \leqslant 0$), while for water ($Pr = 7.00$) higher values of Ri are used without the observation of separation. The boundary layer theory is not applicable for such cases where separation takes place and it is beyond the scope of the present investigation. Therefore, in order to avoid the separation, and to ensure the accuracy of the results in the oscillating plate temperature, the results presented later will consider $Ri = 0.1$ for the opposing flow for both air and water. On the other hand for aiding mixed convection, the present formulation is not applicable for high values of Ri, where the free convection mode is dominated. Therefore, the results presented for the aiding flow for both air and water is limited to $Ri = 1.0$. In view of the good agreement observed in Fig. 2 for different values of Richardson number for both heated surface and cooled surface, the author proceeded to generate results for plate temperature oscillation.

The oscillation of the plate temperature is considered now with an amplitude range from $\varepsilon = 0.1$ to $\varepsilon = 0.5$ and frequency range from $\Omega = 0$ to $\Omega = 5$. The mixed convection process starts when the surface temperature increases suddenly from the ambient temperature T_∞ to the average surface temperature $\check{T}_w$. At this time the ratio $Nu_x/\sqrt{Re}$ goes to infinity. Then, when the surface temperature oscillates the ratio $Nu_x/\sqrt{Re}$ is found to oscillate accordingly. For $Ri \neq 0$ the friction coefficient $C_{fx}\sqrt{Re}$ is found to oscillate as well. This oscillation becomes steady periodic oscillation after some periods of oscillation. The steady periodic oscillation is achieved when the amplitude and the average values of the ratio $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ become constant for different periods. The following condition is considered for the steady periodic oscillation:

$$\frac{A(\Re)^p - A(\Re)^{p-1}}{A(\Re)^p} \leqslant 10^{-3} \quad \text{and} \quad \frac{\overline{\Re}^p - \overline{\Re}^{p-1}}{\overline{\Re}^p} \leqslant 10^{-3} \tag{15}$$

where the superscript p is the period number, and $\Re$ can stand for either $Nu_x/\sqrt{Re}$ or $C_{fx}\sqrt{Re}$ and

$$A(\Re) = \frac{1}{2}\big[\mathrm{Max}(\Re) - \mathrm{Min}(\Re)\big] \tag{16}$$

for $\tau_0 \leqslant \tau \leqslant \tau_0 + (2\pi/\Omega)$ and

$$\overline{\Re} = \frac{1}{(2\pi/\Omega)} \int_{\tau_0}^{\tau_0+(2\pi/\Omega)} \Re \, \mathrm{d}\tau \tag{17}$$

The steady periodic oscillation of $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ for air ($Pr = 0.72$) are shown in Fig. 3(a) and (b) respectively at constant frequency $\Omega = 5$ and different values of the amplitude ε. Similar results for water ($Pr = 7.00$) are shown in Fig. 4(a) and (b) respectively. The ultimate periodic oscillations are shown in Figs. 3 and 4 which are achieved after 8 periods of oscillation. The number of periods at which the solution reaches steady periodic state is affected marginally by the Prandtl number and the frequency of the surface temperature oscillation. The results are presented for three different values of Ri corresponding to aiding mixed convection ($Ri = 1$), forced convection ($Ri = 0$) and opposing mixed convection ($Ri = 0.1$). The Nusselt number is observed to oscillate in all the cases for different values of Ri and ε with a small phase change with the surface temperature oscillation. The values of Nusselt number in the aiding mixed convection ($Ri = 1$) is always higher than the forced convection ($Ri = 0$) and the values of Nusselt number for opposing mixed convection is always the lowest. The oscillation of Nusselt number is more intensive (higher amplitude of the Nusselt number variation) for high values of ε and/or Pr. Figs. 3(a) and 4(a) show also that the Nusselt number becomes negative in some instances during the oscillation period. This means that part of the heat gained by the fluid when the surface temperature is high will return back to the wall when its temperature drops. The friction coefficient presented in Figs. 3(b) and 4(b) is found to oscillate also for the mixed convection modes approximately in-phase with the surface temperature oscillation. In the forced convection mode ($Ri = 0$), it is constant ($C_{fx}\sqrt{Re} = 0.331$) and it is independent of the surface temperature oscillation or the Prandtl number. Figs. 3(b) and 4(b) show that, for the mixed convection mode, $C_{fx}\sqrt{Re}$ is found to have higher values for air ($Pr = 0.72$) than that for water ($Pr = 7.00$) for aiding mixed convection ($Ri = 1$) and the trend is reversed for opposing mixed convection ($Ri = 0.1$).

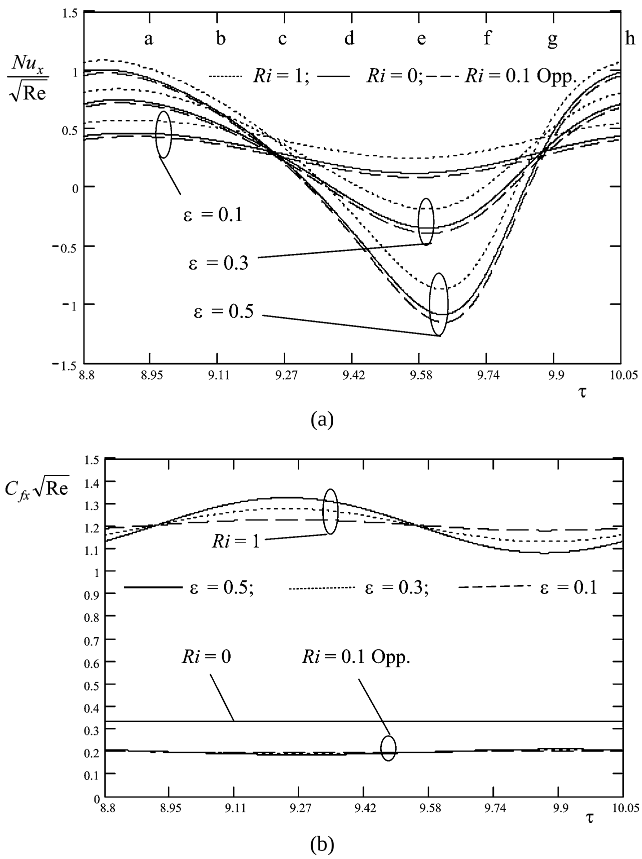


Fig. 3. Periodic oscillation of (a) $Nu_x/\sqrt{Re}$, and (b) $C_{fx}\sqrt{Re}$ with the non-dimensional time for $\Omega = 5$ and $Pr = 0.72$.

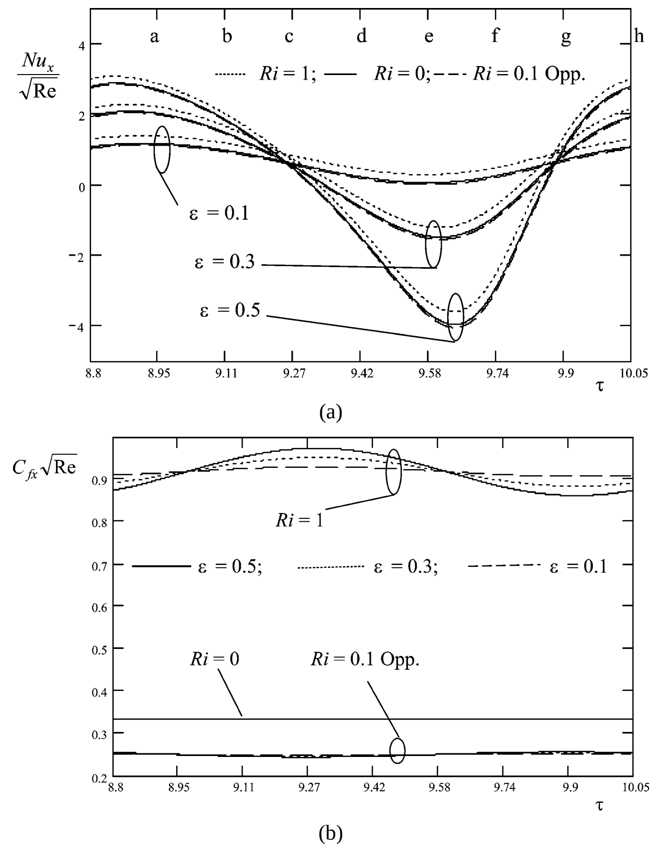


Fig. 4. Periodic oscillation of (a) $Nu_x/\sqrt{Re}$, and (b) $C_{fx}\sqrt{Re}$ with the non-dimensional time for $\Omega = 5$ and $Pr = 7.00$.

To investigate the occurrence of negative Nusselt number, the period of the last cycle is divided into eight time steps (a)–(h), as shown in Figs. 3(a) and 4(a). At each time step the temperature profiles are shown in Figs. 5 and 6 for air and water, respectively, for the periodic aiding mixed convection with $\varepsilon = 0.5$ and $\Omega = 5$. For forced convection and opposing mixed convection, similar temperature profiles are expected, but they are not shown for brevity. It can be seen from Figs. 5 and 6 that the surface temperature is equal at the points (a,c), (d,h) and (e,g) but the temperature gradients at these points are different which gives different values of the Nusselt number. The maximum value of $Nu_x/\sqrt{Re}$ occurs near point (h) when the surface temperature increases from its minimum value to its average value. The minimum value of $Nu_x/\sqrt{Re}$ is negative and it occurs between points (e) and (f) when the surface temperature goes to its minimum value. Negative values of $Nu_x/\sqrt{Re}$ means that there will be some point in the boundary layer with temperature higher than the surface temperature from which heat will transfer partly to the plate. A similar phenomenon is observed for the water with more intensive oscillation of $Nu_x/\sqrt{Re}$ as shown in Fig. 6. It has also been observed from Figs. 5 and 6 that the boundary layer thickness for the air is more than that for water for the surface temperature oscillation as well as in the isothermal case.

The effect of the frequency of the surface temperature oscillation on the Nusselt number oscillation is shown in Fig. 7(a) for $\varepsilon = 0.3$ and different values of Ri and different values of Ω in the ultimate steady period $\Omega\tau$ for $Pr = 7.00$. For air ($Pr = 0.72$) similar trends are observed for the oscillation of $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ but they are not shown for brevity. The Nusselt number oscillates with low amplitude of the Nusselt number variation for small Ω as shown in Fig. 7(a) and vice versa. Fig. 7(b) show the variation of the friction coefficient with the period $\Omega\tau$ of the surface temperature oscillation for $\varepsilon = 0.3$ and different values of Ω. It is observed that for forced convection mode, there is no effect of the frequency of the surface temperature oscillation on the friction coefficient. In the aiding mixed convection mode, it is found that, as Ω increases, the maximum value of $C_{fx}\sqrt{Re}$ is delayed progressively and the amplitude of the friction coefficient oscillation decreases. In the opposing mixed convection mode, the oscillation of the friction coefficient is out of phase with that for the aiding mixed convection as shown in Fig. 7(b).

The cyclic average values of the ratio $Nu_x/\sqrt{Re}$ and $C_{fx}\sqrt{Re}$ were found for the steady periodic state for different values of the amplitude and frequency of the surface temperature oscillation. Figs. 8–10 show the variation of $\overline{Nu_x}/\sqrt{Re}$ with the frequency of the surface temperature oscillation for both air ($Pr = 0.72$) and water ($Pr = 7.00$) for

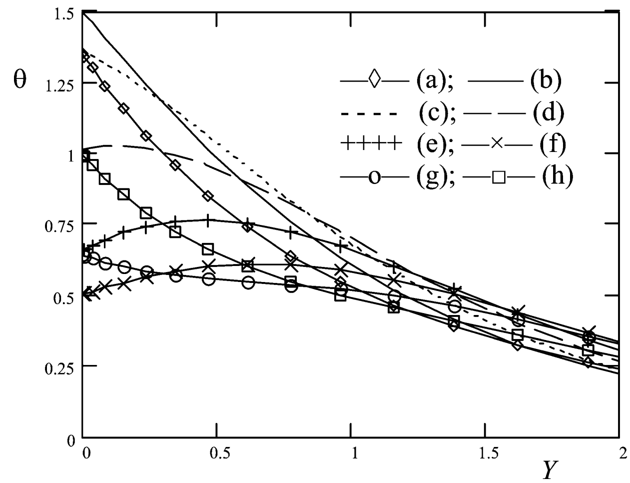


Fig. 5. Periodic state temperature profiles at different time steps (a)–(h) in Fig. 3(a) for the last cycle, $\varepsilon = 0.5$, $\Omega = 5$, $Ri = 1$ and $Pr = 0.72$.

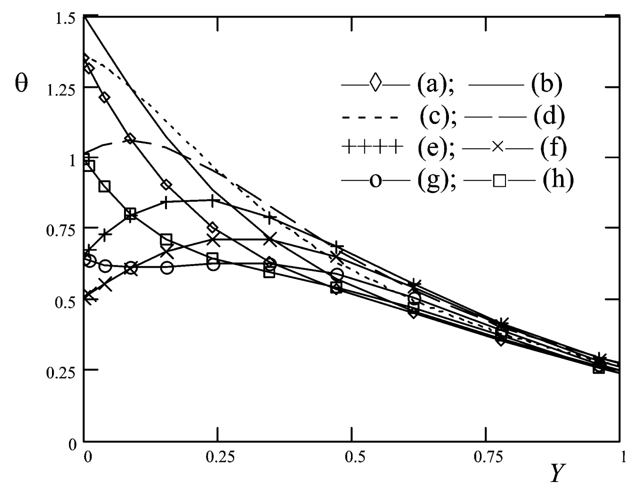


Fig. 6. Periodic state temperature profiles at different time steps (a)–(h) in Fig. 4(a) for the last cycle, $\varepsilon = 0.5$, $\Omega = 5$, $Ri = 1$ and $Pr = 7.00$.

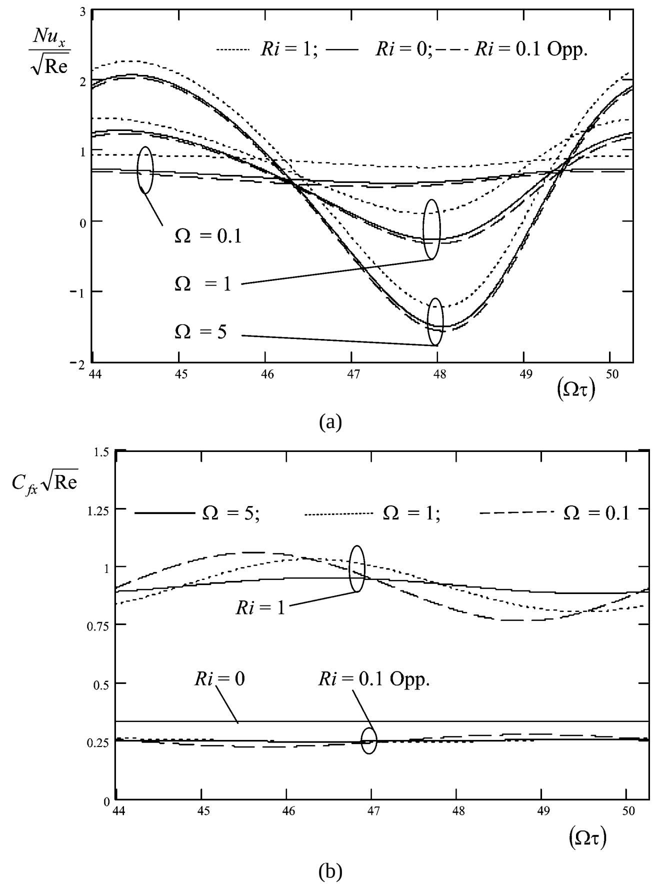


Fig. 7. Periodic oscillation of (a) $Nu_x/\sqrt{Re}$, and (b) $C_{fx}\sqrt{Re}$ with the period of oscillation $(\Omega\tau)$ for $\varepsilon = 0.3$ and $Pr = 7.00$.

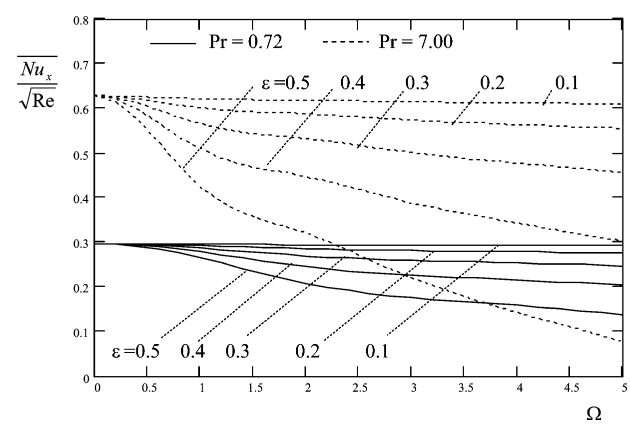


Fig. 8. Variation of $\overline{Nu_x}/\sqrt{Re}$ with the frequency of the surface temperature oscillation at the steady periodic state, forced convection $(Ri = 0)$.

forced convection, aiding mixed convection and opposing mixed convection respectively. While the calculated values of $\overline{C_{fx}}\sqrt{Re}$ for all the cases are found to be constant and approximately equal to the values for non-oscillating surface temperature (for $\varepsilon = 0$) shown in Fig. 2, which indicates insignificant effect of the surface temperature oscillation on the friction coefficient. It can be shown from Figs. 8–10 that, the values of $\overline{Nu_x}/\sqrt{Re}$ decrease when the surface temperature oscillates at higher amplitudes and/or higher frequency of the surface temperature oscillation for both air and water.

The temporal average value of $\overline{Nu_x}/\sqrt{Re}$ can approaches zero at high amplitude and frequency of the surface temperature oscillation. This indicates that most of the heat is oscillating from the surface and some point in the boundary layer without proceeding to the ambient. The results presented in Figs. 8–10 for maximum values of $\varepsilon = 0.5$ and $\Omega = 5$ where the convergence solution is obtained. Difficulties are encountered in getting converged solution from the boundary layer equations for higher values of the amplitude and frequency of the surface temperature oscillations. For forced convection $(Ri = 0)$ and opposing mixed convection $(Ri = 0.1)$ it is observed from Figs. 8 and 10 that, when $\Omega > 3.5$ and $\varepsilon = 0.5$ the value of $\overline{Nu_x}/\sqrt{Re}$ for the steady periodic state for water is less than that of air, which is an unusual behavior.

While for aiding mixed convection, the value of $\overline{Nu_x}/\sqrt{Re}$ for the steady periodic state for water is always greater

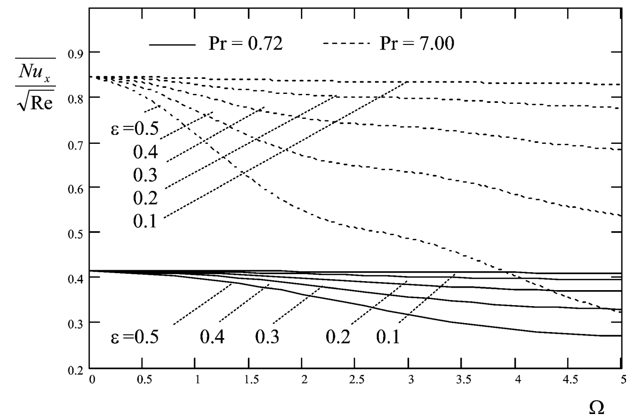


Fig. 9. Variation of $\overline{Nu_x}/\sqrt{Re}$ with the frequency of the surface temperature oscillation at the steady periodic state, aiding mixed convection ($Ri = 1$).

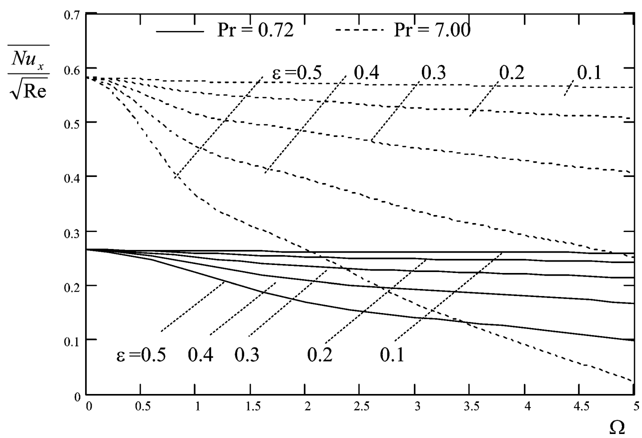


Fig. 10. Variation of $\overline{Nu_x}/\sqrt{Re}$ with the frequency of the surface temperature oscillation at the steady periodic state, opposing mixed convection ($Ri = 0.1$).

than that of air for a given values of Ω and ε as shown in Fig. 9.

5. Conclusions

The present investigation deals with periodic laminar forced convection, aiding mixed convection and opposing mixed convection from a vertical plate. The periodic convection occurs when there is periodic oscillation of the surface temperature. The plate temperature is assumed to be a sinusoidal function with the time, having amplitude and frequency. Two different Prandtl numbers are considered, which are $Pr = 0.72$ (air) and $Pr = 7.00$ (water). In the mathematical formulation of this problem the boundary layer and Boussinesq approximations are used to simplify the problem. The dimensionless forms of the governing equations are solved numerically using a finite-difference method. The initial condition is generated by solving the continuity and momentum equations at steady state. The resultant initial condition is very close to the Blasius boundary layer solution. The developed algorithm is tested for forced and mixed convection and the results show good agreement with the numerical and experimental results in the literature. It has been shown that the Nusselt number oscillates as a result of an oscillating surface temperature and for some cases the Nusselt number becomes negative. For mixed convection modes the friction coefficient also oscillates as a result of oscillating surface temperature. The results are presented to show the effects of the amplitude and the frequency of the plate temperature oscillation on the Nusselt number and friction coefficient in the steady periodic state. The temporal average values of the Nusselt number and friction coefficient are calculated during the period of oscillation of the plate temperature in the steady periodic state. The results also show that the temporal average value of the Nusselt number decreases with increasing the amplitude and/or the frequency of the oscillating surface temperature for both air and water for forced, aiding mixed convection and opposing mixed convection. However, the calculated values of the temporal average value of friction coefficient, for all the cases considered, are found to be constant and approximately equal to the values for non-oscillating surface temperature.

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